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Radar Echo Variations  
of a Large Rough Sphere

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

RADAR ECHO VARIATIONS OF A LARGE ROUGH SPHERE

*M. L. BURROWS*

*Group 61*

TECHNICAL NOTE 1965-46

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## ABSTRACT

The effect of surface roughness on the radar backscattering cross section of a perfectly conducting nominally spherical target is examined by applying the Kirchhoff method.

It is shown that, for the type of roughness and sphere size to which the Kirchhoff method is applicable, the standard deviation of the cross section increases with frequency according to the law  $2\sqrt{2}\sigma_0 k\zeta$  until the first Fresnel zone reduces in size to the scale length of the roughness. At this point a knee in the curve occurs and its further course is determined by a more detailed statistical description of the surface. Here  $\sigma_0$  is the nominal cross section,  $\zeta$  is the standard deviation of the surface height  $h$  and  $k = 2\pi/\lambda$ , where  $\lambda$  is the wavelength.

The average cross section is shown to be given by  $\sigma_0 \left[ 1 + O\{(kh)^3\} \right]$ . In this connection, an error that may be significant, occurring in the work of Hiatt et al. on roughness effects, is pointed out.

Some experimental results are reported which support the theoretical conclusions and, moreover, indicate that they may be useful even when the scale length of the roughness is smaller than the wavelength.

Further theoretical results are included concerning the effect of roughness on a general second order surface and the correlation function for the cross section.

Accepted for the Air Force  
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Lt Colonel, USAF  
Chief, Lincoln Laboratory Office

## Radar Echo Variations of a Large Rough Sphere

### I. INTRODUCTION

Variation with aspect of the radar backscattering cross section of a nominally spherical metal body detracts from its usefulness as a standard of cross section. The situation is deteriorated further if the mean cross section is appreciably different from the nominal cross section. For manufacturing purposes, therefore, it is important to know how the cross section depends upon the surface tolerance.

The effect of roughness having a scale length short compared with a wavelength has been discussed by Hiatt et al (1960) by a method based on an impedance boundary condition. Their theoretical work is limited to finding an expression for the average backscattered field strength, although useful experimental results are presented concerning the variation of the cross section.

At the other extreme of the scale length of the surface roughness, that is, for perturbations which smoothly deform the sphere into some still-convex body like an ellipsoid, for example, the problem is easily tackled by applying the well known relationship between the cross section and the radii of curvature at the specular point.

This paper presents the results of an application of the Kirchhoff method. The basic assumptions made are that the perpendicular distance from the mean surface to the true surface is small compared with a wavelength and that the

scale length of the roughness is small compared with the radius of the sphere but large compared with the wavelength. It is further assumed that the surface is statistically uniform and isotropic.

The scale length assumption, as it stands, limits the application of the results for practical purposes to spheres which have had the small scale roughness removed by polishing. However, experience with the Kirchhoff method has shown that it yields accurate results even when the scale length assumption is relaxed considerably (Bechmann and Spizzichino, 1963:IV). In Section VII some experimental results for rough spheres are reproduced which tend to support this view.

The Kirchhoff approach reduces the problem to a form which has already received considerable attention in the random phase-screen theory of radio-star scintillations induced by the ionosphere, [e.g. Booker, et al (1950), Mercier (1962)].

## II. THE KIRCHHOFF INTEGRAL

The scattering properties of a perfectly conducting sphere can be characterized by the parameter  $ka$ , where  $a$  is the radius of the sphere and  $k = 2\pi/\lambda$ ,  $\lambda$  being the wavelength of the incident radiation. For large values of  $ka$  the backscattered field can be expressed as the sum of an optics terms and a creeping-wave term. For values of  $ka$  larger than about 10, the creeping-wave contribution is less than one tenth that of the optics term (Senior, 1964). Since, in addition, the creeping waves traverse a path on the surface which

is at least  $\pi a$  in length, it is to be expected that the effect of the surface perturbations will average out, in some sense, thereby making the creeping-wave term significantly less sensitive to the roughness than is the optics term. It is reasonable to suppose, therefore, that the effect of neglecting the creeping-wave term will not be serious for even smaller values of  $ka$  than 10.

The optics term, to the first order in  $ka$ , is that obtained by the Kirchhoff method of calculating the scattered field. (If the spurious shadow region contribution is discarded, the Kirchhoff solution is accurate to at least the third order in  $ka$ .)

When a linearly polarized plane wave of unit intensity propagating in the negative  $z$  direction is incident upon the rough sphere, the Kirchhoff solution for the backscattered electric field at a point in the far zone can be written as

$$E = \frac{k}{2\pi R} \int_A \exp\{-i2kz(x, y)\} dy \quad (1)$$

Here  $R$  is the distance from the sphere to the field point,  $z(x, y)$  is the height of the illuminated hemisphere from the plane  $z = 0$  and the integral is carried out over the projected area of the illuminated hemisphere on the plane  $z = 0$ . A superfluous constant phase factor has been omitted and  $E$  is understood to be polarized in the same direction as the incident electric field.

### III. THE VARIANCE OF THE CROSS SECTION

The effect of the surface roughness may be introduced by writing  $z(x, y)$



as

$$z(x, y) = z_o(x, y) + \delta(x, y) \quad (2)$$

where  $\delta$  is the height of the true surface, in the  $z$ -direction, from the mean surface  $z_o(x, y)$ . Thus, by definition,

$$\langle \delta(x, y) \rangle = 0 \quad (3)$$

where the angle brackets denote the operation of taking the average or expected value of the enclosed quantity, and  $z_o(x, y)$  can be written

$$z_o(x, y) = \sqrt{a^2 - x^2 - y^2} - a \quad (4)$$

From (1) and (2), the expected value of  $E$  is given by

$$\langle E \rangle = \frac{k}{2\pi R} \int_A e^{-i2kz_o(x, y)} \langle e^{-i2k\delta(x, y)} \rangle dx dy \quad (5)$$

Figure 1 shows that, to a first approximation,  $\delta(x, y)$  is related to  $h(x, y)$  (the height, in the radial direction, of the true surface from the mean surface) by the equation  $\delta(x, y) = h(x, y)/\cos\theta$ , where  $\theta$  is the angle between the  $z$  axis and the radial direction. Therefore, since the roughness is statistically uniform over the surface, then in the vicinity of the specular point  $\langle \exp\{-i2k\delta(x, y)\} \rangle$  is a weak function of the single argument  $x^2 + y^2$ . In addition, a basic assumption here is that  $h \ll \lambda$ , so that  $2k\delta(x, y)$  is uniformly bounded for all  $k$ . This clears the way for a straightforward stationary phase evaluation of (5), using

the form for  $z_o$  given by (4) with  $ka$  as the large parameter.

The result is  $\langle E \rangle = ia \langle \exp\{-i2k\delta(0, 0)\} \rangle / 2R$ , which, since  $\delta(0, 0) = h(0, 0)$ ,  $h/\lambda \ll 1$  and  $\langle h(0, 0) \rangle = 0$ , may be written

$$\langle E \rangle = (1 - 2k^2 \zeta^2) i \frac{a}{2R} \quad (6)$$

where  $\zeta^2 = \langle h^2(x, y) \rangle$ , or, alternatively,

$$\langle E \rangle = (1 - 2k^2 \zeta^2) E_o, \quad (7)$$

where  $E_o$  is the backscattered field of the perfect sphere of radius  $a$ .

It is shown in the Appendix that, for small perturbations of the scattered field, the normalized variance  $D\{\sigma\}$  of the cross section  $\sigma$  is given by

$$D\{\sigma\} \equiv \left\langle \left[ \frac{\sigma - \langle \sigma \rangle}{\langle \sigma \rangle} \right]^2 \right\rangle = 2 \left\langle \left| \frac{E - \langle E \rangle}{\langle E \rangle} \right|^2 \right\rangle + \text{Re} \left\{ \left[ \frac{E - \langle E \rangle}{\langle E \rangle} \right]^2 \right\} \rangle, \quad (8)$$

where  $\text{Re}$  denotes the operation of taking the real part. Therefore, from (1), (2), (5) and (6), this may be written, to  $O\{(k\zeta)^2\}$ , as

$$D\{\sigma\} = \frac{2k^2}{\pi a^2} \left[ \int e^{-i2k[z_o(x, y) - z_o(x', y')]} F_1(x, y, x', y') dx dy dx' dy' \right. \\ \left. - \text{Re} \left\{ \int e^{-i2k[z_o(x, y) + z_o(x', y')]} F_2(x, y, x', y') dx dy dx' dy' \right\} \right], \quad (9)$$

where

$$F_1(x, y, x', y') = \langle e^{-i2k[\delta(x, y) - \delta(x', y')]} \rangle - \langle e^{-i2k\delta(x, y)} \rangle \langle e^{i2k\delta(x', y')} \rangle$$

$$F_2(x, y, x', y') = \langle e^{-i2k[\delta(x, y) + \delta(x', y')]} \rangle - \langle e^{-i2k\delta(x, y)} \rangle \langle e^{-i2k\delta(x', y')} \rangle .$$

It is clear that both  $F_1$  and  $F_2$  are zero when the distance between the points  $(x, y)$  and  $(x', y')$  is large enough for the corresponding  $\delta$ 's to be uncorrelated. Therefore, when the scale length  $L$  of the surface roughness is smaller than the width  $w$  of the first Fresnel zone, the variation of  $F_1$  and  $F_2$  as functions of  $x - x'$ ,  $y - y'$  is stronger than that of the exponential factors and the method of stationary phase is not directly applicable.

However, the period of the exponential factors in the integrands in (9) steadily increases as  $x, x', y$  or  $y'$  increase, so that eventually the period is small compared with the scale length of the roughness. Such regions contribute negligibly to the integrals, which leads to the conclusion that the effective region of integration in  $x, x', y, y'$  space is a hyper-sphere centered at the origin of radius of order  $r_0$ , where  $r_0$  is the value of  $\sqrt{x^2 + y^2}$  at which the period of the exponential factors is equal to the scale length  $L \sqrt{1 - (x^2 + y^2)/a^2}$  of the roughness. (The scale length on the  $x, y$  plane is a projection of the true surface scale length.) The value of  $r_0$  is easily deduced to be  $a\lambda/4L$ . Since, by assumption,  $\lambda/L \ll 1$ ,  $r_0$  is small compared with the radius of the

sphere.

Therefore, for all  $L$ , the significant contribution to the integrals in (9) derives from a small region about the origin within which  $\delta(x, y)$  and  $z_0(x, y)$  are given essentially by  $h(x, y)$  and  $-(x^2 + y^2)/2a$ , respectively. In addition, since the limits of integration do not lie within or close to this region, they may be extended to infinity.  $D\{\sigma\}$  can then be written as

$$D\{\sigma\} = \frac{8k^4 \zeta^2}{\pi^2 a^2} \left[ \int e^{i \frac{k}{a} [x^2 + y^2 - x'^2 - y'^2]} C_h \left( \sqrt{(x - x')^2 + (y - y')^2} \right) dx dx' dy dy' \right. \\ \left. + R_e \left\{ \int e^{i \frac{k}{a} [x^2 + y^2 + x'^2 + y'^2]} C_h \left( \sqrt{(x - x')^2 + (y - y')^2} \right) dx dx' dy dy' \right\} \right], \quad (10)$$

where  $\zeta^2 C_h \left( \sqrt{(x - x')^2 + (y - y')^2} \right) = \langle h(x, y) h(x', y') \rangle$ ,  $C_h(0) = 1$  and the fact that  $h/\lambda \ll 1$  has been used to reduce  $F_1$  and  $F_2$  to the expressions  $4k^2 \zeta^2 C_h$  and  $-4k^2 \zeta^2 C_h$  respectively. As defined here  $C_h$  is the correlation function for the surface.

By means of the substitutions  $x = u + v/2$ ,  $x' = u - v/2$ ,  $y = u' + v'/2$ ,  $y' = u' - v'/2$ , the form for  $D\{\sigma\}$  given by (10) simplifies to

$$D\{\sigma\} = 8k^2 \zeta^2 \left[ 1 - \int_0^\infty C_h(\sqrt{2ap/k}) \sin p \, dp \right] . \quad (11)$$

This is the required result.

When the scale length is small compared with the width  $w$  of the first Fresnel zone ( $w = \sqrt{2\pi a/k}$ ), the integral in (11) is clearly small compared with unity. Thus in this, the case of most practical interest, the normalized standard deviation  $\sqrt{D\{\sigma\}}$  of the cross section is given simply by  $2\sqrt{2k}\zeta$ .

#### IV. THE AVERAGE CROSS SECTION

The cross section is proportional to  $|E|^2$ , so that the average normalized cross section  $\langle\sigma\rangle/\sigma_0$  is given by

$$\frac{\langle\sigma\rangle}{\sigma_0} = \frac{\langle |E|^2 \rangle}{|E_0|^2} = \frac{|\langle E \rangle|^2}{|E_0|^2} \left\{ 1 + \left\langle \left| \frac{E - \langle E \rangle}{\langle E \rangle} \right|^2 \right\rangle \right\} \quad (12)$$

The rearrangement of the right side includes the term  $\langle |(E - \langle E \rangle)/\langle E \rangle|^2 \rangle$  which has been evaluated in Section III. It is, from (8), just half the first term on the right of (11), or  $4k^2 \zeta^2$ . Therefore, (12) can be rewritten, using (7), as  $\langle\sigma\rangle/\sigma_0 = (1 - 2k^2 \zeta^2)^2 (1 + 4k^2 \zeta^2)$ , which is unity to the order in  $kh$  to which the approximation has been carried. That is

$$\frac{\langle\sigma\rangle}{\sigma_0} = 1 + 0 \left\{ (kh)^3 \right\} . \quad (13)$$

This result leads one to conclude that a departure of the average cross section from the nominal cross section is unlikely to be measureable under the conditions for which the theory is valid.

It is of interest at this point to mention the work of Hiatt et al (1960), who use an impedance boundary condition to represent small scale surface roughness. They find an expression for the expected value of the backscattered field and then square this to obtain the expected value of the backscattering cross section. This is clearly in error, for  $\langle |E|^2 \rangle = \langle | \langle E \rangle + (E - \langle E \rangle) |^2 \rangle = \langle \langle E \rangle |^2 \rangle + \langle |E - \langle E \rangle|^2 \rangle$ , so that they have neglected to include the term  $\langle |E - \langle E \rangle|^2 \rangle$ .

The application of the Kirchhoff method to the same problem has shown that when this additional term is included it just cancels the effect of the roughness on the term  $|\langle E \rangle|^2$ . Hence, although the two approaches to the problem do not start from the same assumptions, the Kirchhoff method indicates the necessity of including the additional term. One concludes, therefore, that a modification of the average backscattering cross section due to small scale roughness has yet to be demonstrated theoretically.

## V. GENERALIZATIONS

It is a simple matter to rework the derivation for a mean surface in the form of a general second-order surface. Instead of the single radius  $a$ , one has to deal with the separate principle radii of curvature  $a_1$  and  $a_2$  at the specular point.

In place of (11), one finds

$$D\{\sigma\} = 8k^2 \zeta^2 \left[ 1 - \frac{1}{2\pi} \int_0^{2\pi} \int_0^\infty C_h \left\{ \sqrt{2(a_1 \cos^2 \theta + a_2 \sin^2 \theta) \rho / k} \right\} \sin \rho \, d\rho \, d\theta \right] ,$$

so that again, when the scale length is small compared with the least width of the first Fresnel zone, the integral is small and the normalized standard deviation of the cross section is  $2\sqrt{2k}\zeta$ .

Equation (13) for the average cross section is found to apply unaltered to the more general surface.

A further result of some interest is the correlation function  $C_\sigma(\theta)$  for the backscattering cross section, which may be defined as

$$C_\sigma(\theta) = \frac{R(\theta)}{R(0)} \quad (14)$$

where  $R(\theta) = \langle \sigma(\theta_0) \sigma(\theta_0 + \theta) \rangle - \langle \sigma \rangle^2$  and the arguments  $\theta_0$  and  $\theta_0 + \theta$  specify the directions, based on an arbitrary great circle on the surface of the sphere, at which the cross sections are measured.

The derivation is straightforward provided  $a\theta$  is small compared with the radius of the sphere, for then it is sufficient to replace  $x'$  and  $y'$  in the exponential factors in the integrands in (9) by  $x' - x_0$  and  $y' - y_0$ , where  $x_0^2 + y_0^2 = a^2 \theta^2$ . The result may be expressed as

$$R(\theta) = 8k^2 \zeta_o^2 \sigma_o^2 \left[ C_h(a\theta) - \int_0^\infty C_h(\sqrt{2a\rho/k}) J_o(\theta\sqrt{2ka\rho}) \sin\left(\rho + ka\theta^2/2\right) d\rho \right], \quad (15)$$

where  $J_n(u)$  is the Bessel function of the first kind of order  $n$  and argument  $u$ .

When the scale length of the surface perturbations is small compared with the width of the first Fresnel zone, the integral in (15) is clearly small compared with the first term in brackets. In this case (15) reduces to  $R(\theta) = 8k^2 \zeta_o^2 \sigma_o^2 C_h(a\theta)$ , so that the correlation function for the cross section is, from (14), given simply by  $C_\sigma(\theta) = C_h(a\theta)$ .

## VI. NUMERICAL STUDY

The derivation of expression (11) for the variance of the cross section is based on the well tried Kirchhoff method. Thus if the result is incorrect it is likely to be because the derivation is faulty rather than the Kirchhoff method.

To provide a check on the derivation, a computer program was written to compute, using the Kirchhoff method, the backscattering cross section for a sphere with a particular type of random surface. By repeating the computation for a number of different samples of the same random process the variance of the cross section was obtained.

The random surface was generated by dividing the surface of the mean sphere into a large number of curvilinear squares of roughly equal area. The true surface was then defined to be given in spherical coordinates by



$r(\theta, \phi) = a + h(\theta, \phi)$ , with  $h(\theta, \phi)$  being constant for all points  $(\theta, \phi)$  lying within a particular curvilinear square. The value of  $h$  assigned to each square was an independent sample of a Gaussian random process of zero mean and given variance.

The results are shown in Table I. The entries in columns 1 – 5 give the various ratios of significance to the computation, with  $L$  being taken as twice the side length of the elemental surface areas. The entries in column 6 are the corresponding values of the normalized standard deviation of the cross section given by the expression  $2\sqrt{2}k\zeta$ , which, according to the derivation is appropriate for small values of  $L/w$ . The normalized standard deviation of the  $N$  cross section values obtained with the computer program is entered in column 7 and  $N$  is recorded in column 8.

TABLE I

COMPUTATION OF THE NORMALIZED STANDARD  
DEVIATION OF CROSS SECTION FOR A ROUGH SPHERE

1	2	3	4	5	6	7	8
$ka$	$\zeta/\lambda$	$L/\lambda$	$L/a$	$(L/w)^2$	$2\sqrt{2}k\zeta$	$\sqrt{D}\{\sigma\}$	$N$
50	$0.8 \times 10^{-3}$	2.2	0.28	0.65	0.014	0.017	19
100	$1.6 \times 10^{-3}$	4.6	0.28	1.3	0.028	0.029	19
200	$3.2 \times 10^{-3}$	9.1	0.28	2.6	0.056	0.031	19
200	$3.2 \times 10^{-3}$	2.2	0.072	0.16	0.056	0.051	12

The first three rows show the effect of increasing the frequency, leaving all else unchanged. The agreement between columns 6 and 7 is good except at the highest frequency, where, in view of the fact that  $(L/w)^2$  has the value 2.6, it is to be expected that the simple form  $2\sqrt{2}k\zeta$  should over-estimate  $\sqrt{D\{\sigma\}}$ . The final row shows the effect of then reducing  $L$  by a factor of 0.25, which makes  $(L/w)^2$  small again and the agreement is regained.

## VII. EXPERIMENTAL RESULTS

The theoretical work reported here was carried out as a part of the program which culminated in the successful launch of a 113 cm radar calibration sphere (Object number 1361) on May 6, 1965 into a nearly circular 1500 nautical-mile orbit of  $32.11^\circ$  inclination. The initial attempt to manufacture the sphere involved the welding together of two hemispherical aluminum shells. It was found that the welding operation distorted the shells over a wide equatorial zone to a degree that made the resulting sphere unacceptable as a radar calibration device. For this reason, a different method of joining the two hemispheres was used for the flight sphere. However, the welded sphere was useful for surface roughness experiments because the variations in cross section were large enough to enable good measurements to be made on a standard radar cross-section range.

The sphere was polished after welding, so that the final condition of the surface was very smooth with undulations of the order of 0.1 cm in height and 15 cm in length. The surface was not statistically uniform, since the

surface was most "rough" in the region of the equator. Therefore, since this immediately violates one of the conditions necessary for the simple theory, it was decided that a detailed dimensional survey of the surface would not be worthwhile. Accordingly, the two statistical characterizations,  $\zeta$  and  $C_h(\rho)$ , of the surface necessary to evaluate  $D\{\sigma\}$  using (11) were estimated from a single sectional profile of the sphere taken by means of an accurate circular check-ring lying in the equatorial plane (the plane of the weld line). The distance from the check-ring to the surface was measured at  $10^\circ$  increments round the full circle.

The check-ring measurements were reduced to equivalent "displacement-from-mean-sphere" measurements by subtracting from them their mean and their first Fourier component. The standard deviation  $\zeta$  of the reduced measurements was found to be 0.0591 cm and the correlation function (defined to be unity at  $0^\circ$ ) was found to be 0.607 at  $10^\circ$  and -0.017 at  $20^\circ$ . The last number indicates that the correlation function falls quickly towards zero and in addition, since the surface is smooth, the slope of the correlation function must be zero at the origin. This indicates that an appropriate simple model for the correlation function is  $C_h(\rho) = \exp(-\rho^2/L^2)$  with  $L = 13.95$ , the value required to make  $C_h(\rho)$  equal to 0.607 when  $\rho$  is the surface arc length equivalent to an angular displacement of  $10^\circ$ .

Substituting this form for  $C_h(\rho)$  into (11) leads to the following expression for the standard deviation of the backscattering cross section.

$$\sqrt{D\{\sigma\}} = \frac{2\sqrt{2k}\zeta}{\sqrt{1 + \left(\pi \frac{L^2}{w^2}\right)^2}} , \quad (16)$$

where  $w = \sqrt{a\lambda}$  is the width of the first Fresnel zone.

Figure 2 is a graph of expression (14) as a function of frequency using the values of  $\zeta$  and  $L$  obtained as described from the check-ring measurements and the nominal sphere radius of 56.5 cm. The broken straight line is the low frequency asymptote.

The measured values of the standard deviation of the backscattering cross section are entered at points. Measurements were made at UHF and in the bands L, S, X, and K. The plotted points were computed from the 360° range record taken around the equator or weld line of the sphere.

In Table II the theoretical and experimental data are presented numerically.

TABLE II  
COMPARISON OF THEORY AND  
EXPERIMENT FOR 113 cm SPHERE

Frequency (GHz)	ka	Wavelength (cm)	$\sqrt{D}\{\sigma\}$		
			Theoretical	Measured	Measured (db)
0.4	4.73	75	0.0137	0.018	0.078
1.2	14.2	25	0.0385	0.046	0.20
3.3	39.1	9.09	0.0742	0.064	0.28
8.5	100.6	3.53	0.0923	0.087	0.38
24.0	284	1.25	0.0965	0.187	0.86

The standard deviation of the measured values is also expressed in decibels for convenience.

The agreement between theory and experiment is surprisingly good in view of the approximations used to describe the surface. The fact that at 24 GHz, the agreement is lost is probably due to the coarse measuring interval ( $10^\circ$ ) used to plot the sectional profile. For at 24 GHz the first Fresnel zone is about  $8.5^\circ$  wide, so at this frequency, smaller scale roughness, undisclosed by the coarse measuring interval, comes into play.

It should be mentioned that at the three higher frequencies it was possible to expand the scale factor of the output pen recording to a value typically in the region of 1 db/in. The equipment used at 0.4 and 1.2 GHz, however, was not amenable to this modification with the result that the scale factor

remained at about 6 db/in. Thus the figure of 0.078 db for the standard deviation of the cross section at 0.4 GHz was obtained from a pen recording with a peak-to-peak variation of about 0.05 inch. In addition, the background level during the measurement was of the order of -40 db, which is too high to be negligible. Therefore, the apparent agreement at 0.4 GHz is likely to be more fortuitous than real.

It is interesting to note that at the two lowest measurement frequencies the wavelength was larger than the scale length of the roughness. This violates one of the conditions required by the Kirchhoff method, but there is no apparent divergence between theory and experiment as the wavelength increases.

Some further experimental data of interest are contained in the paper by Hiatt et al. (1960). They measured the standard deviation of the cross section of a rough 26 cm aluminum sphere at two wavelengths, 3.1 cm and 1.3 cm, and obtained the figures of 0.4 db and 1.04 db, respectively. The standard deviation of the true surface from the mean surface was estimated as 0.037 cm and the scale length of the roughness as 0.101 cm. Thus at both frequencies the scale length to wavelength ratio was less than one tenth, which places the character of the roughness well away from the regime of applicability of the Kirchhoff method. However, a formal application of (11) to this case yields the figures of 0.92 db and 2.2 db for the standard deviation of the cross section at the wavelengths 3.1 cm and 1.3 cm, respectively. (For such a

small scale length, the abbreviated form of (11) applies, making it unnecessary to know the form of the correlation function for the roughness.)

A second assumption necessary to derive (11) was that the maximum perturbation of the cross section be small. This, too, is not true for the present case, and additional error is incurred in assuming a linear relation between the standard deviation expressed as a pure number and the same quantity when expressed in decibels. In spite of this, the theory agrees with experiment to within a factor of about two at both measurement frequencies. This suggests that (11) may be a useful guide to the effect of roughness even for scale lengths considerably smaller than a wavelength, but further measurements using smaller amplitude roughness are necessary to find out whether this is so.

There was no possibility of measuring the average departure of the cross section from nominal, because the sphere itself was at least as good, for the purposes of calibration, as any other available standard.

## VIII. CONCLUSIONS

The effect of surface roughness on the radar backscattering cross section of a large metal sphere has been examined by applying the Kirchhoff method. The roughness has been assumed to be statistically uniform and isotropic, and the effect on the cross section has been determined by obtaining expressions for the average cross section and the standard deviation of the cross section.

It is difficult to prepare spheres having rough surfaces with accurately

known and uniform statistics, and equally difficult to carry out accurate measurements on the small perturbations of the cross section, but those experimental results which have been obtained appear to confirm the validity of the theory within its region of applicability and to suggest that the theory may be a useful guide even for roughness of scale size small compared a wavelength.



## APPENDIX

This appendix records the derivation of (8) in the main text.

The backscattered complex field  $E$  may be written as

$$E = \langle E \rangle + \delta E \quad (\text{A-1})$$

and if it be assumed that  $|\delta E| \ll |E|$ , then the absolute value  $|E|$  of  $E$  is given closely by the absolute value  $|\langle E \rangle|$  of  $\langle E \rangle$  together with the component of  $\delta E$  in the direction of  $\langle E \rangle$ . That is

$$|E| = |\langle E \rangle| + \text{Re} \left\{ \frac{\delta E \langle E \rangle^*}{|\langle E \rangle|} \right\} \quad (\text{A-2})$$

where the star denotes complex conjugate.

From (A-1) it is clear that  $\langle \delta E \rangle = 0$ , so that by taking the expected value of (A-2) one finds  $\langle |E| \rangle \approx |\langle E \rangle|$  which enables one to rewrite (A-2) as

$$|E| - \langle |E| \rangle \approx \text{Re} \left\{ \frac{\delta E \langle E \rangle^*}{|\langle E \rangle|} \right\} .$$

Therefore,

$$\begin{aligned} \frac{|E| - \langle |E| \rangle}{\langle |E| \rangle} &\approx \text{Re} \left\{ \frac{\delta E \langle E \rangle^*}{|\langle E \rangle|^2} \right\} \\ &\approx \text{Re} \left\{ \frac{E - \langle E \rangle}{\langle E \rangle} \right\} \end{aligned} \quad (\text{A-3})$$

since  $|\langle E \rangle|^2 = \langle E \rangle \langle E \rangle^*$  and  $\delta E = E - \langle E \rangle$ .

Now the cross section,  $\sigma$ , is proportioned to  $|E|^2$ , so that

$$D\{\sigma\} \equiv \left\langle \left[ \frac{\sigma - \langle \sigma \rangle}{\langle \sigma \rangle} \right]^2 \right\rangle = \left\langle \left[ \frac{|E|^2 - \langle |E|^2 \rangle}{\langle |E|^2 \rangle} \right]^2 \right\rangle . \quad (\text{A-4})$$

The notation  $\delta |E| = |E| - \langle |E| \rangle$  allows the right side of (A-4) to be expanded and written as

$$\left\langle \left[ \frac{|E|^2 - \langle |E|^2 \rangle}{\langle |E|^2 \rangle} \right]^2 \right\rangle = 4 \frac{\langle \delta |E|^2 \rangle}{\langle |E|^2 \rangle^2} \left[ 1 + 0 \left\{ \frac{\delta |E|}{|E|} \right\} \right] \quad (\text{A-5})$$

so that, since both  $\delta |E| \leq |\delta E|$  and, by assumption,  $|\delta E| \ll |E|$ , the factor in square brackets on the right of (A-5) is essentially unity and

$$D\{\sigma\} \approx 4 \frac{\langle \delta |E|^2 \rangle}{\langle |E|^2 \rangle^2} = 4 \left\langle \left[ \frac{|E| - \langle |E| \rangle}{\langle |E| \rangle} \right]^2 \right\rangle . \quad (\text{A-6})$$

By combining (A-3) with (A-6), the required expression (8) is obtained.

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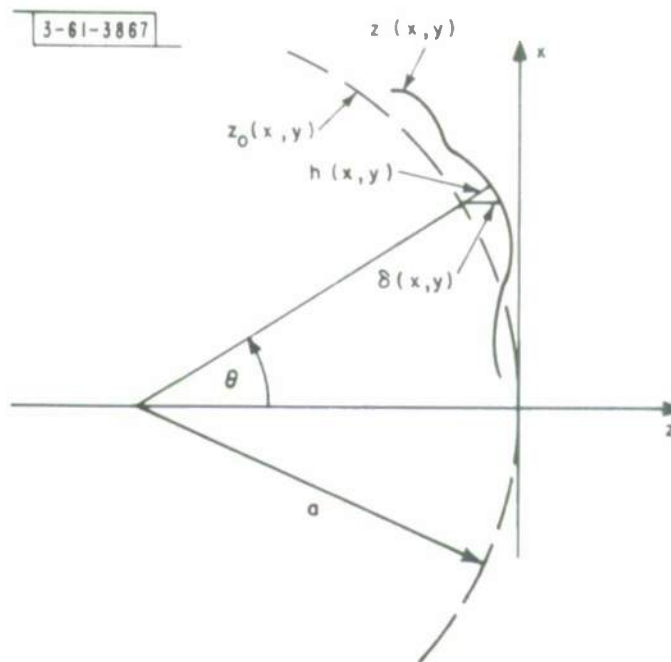


Fig. 1. Geometry of rough sphere.

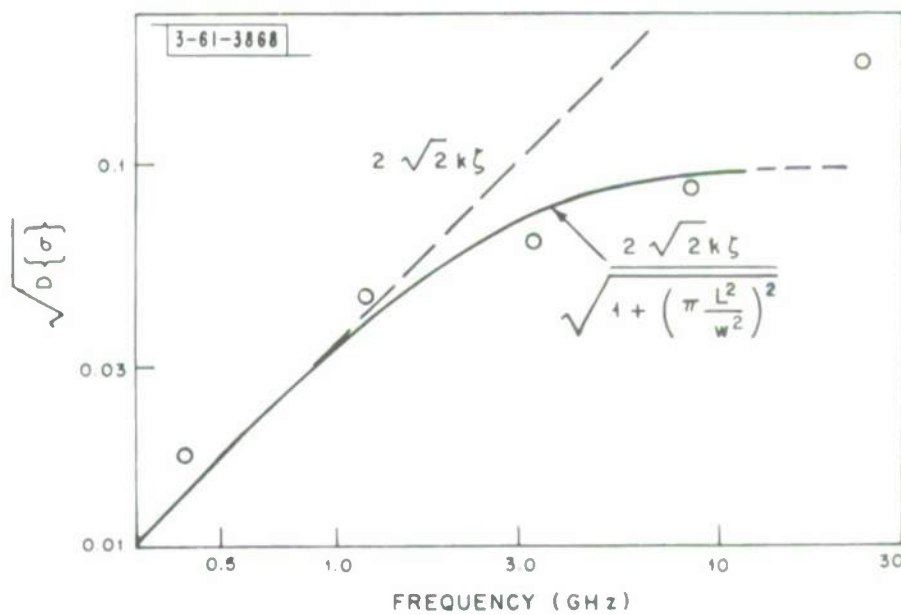


Fig. 2. Standard deviation of echo area as a function of frequency - theory and measurement.



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<p>The effect of surface roughness on the radar backscattering cross section of a perfectly conducting nominally spherical target is examined by applying the Kirchhoff method.</p> <p>It is shown that, for the type of roughness and sphere size to which the Kirchhoff method is applicable, the standard deviation of the cross section increases with frequency according to the law <math>2\sqrt{2}\sigma_0 k \xi</math> until the first Fresnel zone reduces in size to the scale length of the roughness. At this point a knee in the curve occurs and its further course is determined by a more detailed statistical description of the surface. Here <math>\sigma_0</math> is the nominal cross section, <math>\xi</math> is the standard deviation of the surface height <math>h</math> and <math>k = 2\pi/\lambda</math>, where <math>\lambda</math> is the wavelength. The average cross section is shown to be given by <math>\sigma_0 \{1 + O[(kh)^3]\}</math>. In this connection, an error that may be significant, occurring in the work of Hiatt <u>et al</u> on roughness effects, is pointed out.</p> <p>Some experimental results are reported which support the theoretical conclusions and, moreover, indicate that they may be useful even when the scale length of the roughness is smaller than the wavelength. Further theoretical results are included concerning the effect of roughness on a general second order surface and the correlation function for the cross section.</p>		
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